Worcester County Mathematics League

WOCOMAL Varsity Meet #3

Coaches' Booklet

February 2, 2005

Round 1:   Similarity and Pythagorean Theorem

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   ABCD is a parallelogram; on side AB a point E is taken so that AE = ¼ AB. F is
        the midpoint of DC. EF cuts BD at P. Find the ratio of DP to BP.

2. The sides of a right triangle are 3 ft., 4 ft., and 5 ft. in length. A point is taken on
    the hypotenuse at a distance of 2 ft. from the vertex adjacent to the 4 ft. side. If
    the distance from this point to the vertex of the right angle is written in simplified
    radical form as   **a√(b)** **,** what is the value of **(a+b+c)** ?
                             c

3.   Two poles, **p** feet and **q** feet in length are placed **x** feet apart. Lines are drawn
        from the top of each pole to the bottom of the other. The two lines will intersect
        at how many feet above the ground? Assume the poles are perpendicular to a
        piece of level ground. Let the height of the point of intersection be **h** and express
        h, as a single fraction, in terms of only **p** and **q**.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Tahanto, Shepherd Hill, Worcester Academy

Round 2:   **Algebra 1 (open)**

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Vijay hits the golf ball further than Brad, but not as far as Mike. Tiger hits the golf ball somewhat further than Mike, but not as far as John. Who is the third longest hitter of the bunch?

2.   If $x + y = 5$ and $x^2 + 3xy + 2y^2 = 40$, find the value of $(2x + 4y)$.

3.   The difference between two positive numbers is $4\sqrt{3}$. The product of the two numbers is 8. What is the absolute value of the difference of their reciprocals?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Leicester, St. John's

Round 3:   Functions (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   If $P(x) = 2x^3 + kx^2 + x$, find k such that $(x-1)$ is a factor of $P(x)$.

2.   If *F(x)* is a function from the real numbers to the real numbers, we say that *F(x)* has property (I) if $F(F(x)) = x$ for all real numbers x, and we say that *F(x)* has property (J) if $F(-F(x)) = -x$ for all real numbers x. How many linear functions, i.e. functions of the form $F(x) = mx + b$, have both property (I) and property (J)?

3.   Let $g(x) = ax^7 + bx^3 + cx - 7$, where a,b, and c are constants. If $g(-5) = 5$, then find the value of $g(5)$.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Pomfret, Tantasqua, Bromfield,

Round 4:   Combinatorics

ALL ANSWERS MUST BE AS DIRECTED IN THE PROBLEM.

1.   What is the sum of the coefficients of the expression $(a+b)^8$?

2.   My favorite place for ice cream has 23 different flavors. They serve single, double, and triple scoop cones. How many different cones could I buy, if order is important and flavors can be repeated?

3.   Three cans are labeled with "Dimes", "Quarters", and "Half-Dollars". In how many ways can coins be placed in the appropriate cans such that the total value of the coins in the three cans is $20.00 ?

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Bartlett, Bromfield

Round 5:   Analytic Geometry (NO CALCULATORS)

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM.

1.   Find the distance between the origin and the center of the circle whose equation is:

$$x^2 + y^2 - 6x - 4y + 11 = 0$$

2.   A point (x,y) is the circumcenter (center of circumscribed circle) of the triangle with vertices A(-5,-1), B(3,3), and C(5,-5). Find the sum, (x+y).

3.   In the xy-plane there are lines whose x-intercept is a positive prime number and whose y-intercept is a positive integer which pass through the point (4,3). Find the **sum** of the x-intercepts of all such lines.

ANSWERS

(1 pt.)   1. _____

(2 pts.)   2. _____

(3 pts.)   3. _____

Auburn, Shrewsbury

TEAM ROUND

**ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND WRITTEN ON
THE SEPARATE TEAM ANSWER SHEET.   (2 points each)**

1.   The bases of a trapezoid are 7 cm and 13 cm. A line which is twice as far from
        the shorter base as from the longer base divides the figure into two parts. If the
        line is parallel to the bases and the area of the smaller part is 16 cm^2, find the area
        of the larger part in cm^2.

2.   How many different chords are determined by **n** distinct points lying on a circle?

3.   Let $h(x+1) = h(x) + 4/3$ for x = 1,2,3,… and $h(1) = -2/3$. Find $h(3)$.

4.   For what value of **m** will the triangle formed by the lines y = 5, y = **m**x – 6, and
        y = -**m**x – 6, be equilateral?

5.   If the length and width of a rectangle were each increased by 1, the area would be
        84. The area would be 48 if the length and width were each diminished by 1.
        Find the perimeter of the original rectangle.

6.   Given a right triangle with sides of length a,b,and c and area, $a^2 + b^2 – c^2$. Find
        the ratio, c/b, the ratio of the legs of the right triangle.

7.   Given that $f(x) = (x+1)/(x-1)$, solve $f^1 (1/x) = 3$.

8   How many non-congruent isosceles triangles have a perimeter of 16 inches and an
        area of 12 in^2 ?

9.   Find the sum of all positive proper fractions whose denominators are less than or
        equal to 100 (Include unreduced fractions in the sum.).

 Notre Dame Academy, Doherty, Algonquin, West Boylston, Burncoat, Tahanto

Round 1: Similarity and Pythagorean Theorem

1.	(1 pt.)	2/3
2.	(2 pts.)	16
3.	(3 pts.)	$pq/(p+q)$

Round 2: Algebra 1-open

1.	(1 pt.)	Mike
2.	(2 pts.)	16
3.	(3 pts.)	$\frac{1}{2}\sqrt{3}$

Round 3: Functions

1.	(1 pt.)	-3
2.	(2 pts.)	2
3.	(3 pts.)	-19

Round 4: Combinatorics

1.	(1 pt.)	256 or 2^8
2.	(2 pts.)	12719
3.	(3 pts.)	861

Round 5: Analytic Geometry

1.	(1 pt.)	$\sqrt{13}$
2.	(2 pts.)	-13/9
3.	(3 pts.)	12

February 2, 2005

1.   24

2. $(n^2-n)/2$

3.   2

4. $\sqrt{3}$

5.   36

6.   4

7.   1/2

8.   2

9.   2475

TEAM ROUND

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM AND ON THIS SEPARATE TEAM ANSWER SHEET.   (2 points each)

1. _____

2. _____

3. _____

4. _____

5. _____

6. _____

7. _____

8. _____

9. _____

February 2, 2005   **WOCOMAL Varsity Meet**

TEAM ROUND   **School: _____**

Team #:_____

Team Members:

1.   _____

2.   _____

3.   _____

4.   _____

5.   _____